Unveiling the Conceptual Barriers in Geometric Visualization through Newman's Error Analysis

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Abstract

Students are facing difficulties in the use of reasoning and problem solving skills in geometry. It causes significantly low performance in mathematics. The data was collected from sixteen students of grade IX from a community schools in the Kathmandu district. This research aims to investigate the learners' difficulties encountered in the experimental verification of the geometric statements. For this purpose, the answer sheets of sixteen students were divided in four groups according to their errors in proving the geometry statement. After that, researchers randomly selected fours students S_1, S_2, S_3, and S_4 based on the homogenous groups. These answer sheets were analyzed for the exploration of errors in the geometrical reasoning. The students reasoning skills in the proof of the statement 'Midpoint theorem' were analyzing by using Newman's Error Analysis methods. The findings revealed that students struggled in experimental proof of the geometric problem, exhibiting a lack of mastery over the subject matter and an understanding of experimental proof in geometric reasoning. Eventually, the issues of errors of the students themselves, particularly their deficits skills in applying prior concepts.

Keywords: Experimental verification, Mathematical error, Midpoint Theorem, Newman Error Analysis Framework, Geometric reasoning.

Introduction

The research on mathematics education pressures the determination to comprehend the learners' error patterns to update the instructional approaches and enhance in the learning outcomes. The errors of students based on the geometric reasoning were analyzed through the hierarchical framework with Newman's Error Analysis (NEA) framework. The specific cognitive barriers that hinder students' mathematical problem solving processes and targeted in the interventions and addressing the challenges that have been studied. The experimental verification of the statements that ' The line joining the midpoint of two sides of triangle is half of third side'. This fundamental geometric principle reveals the importance of the students' conceptual understanding of relationships of the parallel lines in triangles, and their ability to concentrate theoretical knowledge into experimental verification.

Statement of Problem

Although geometric concepts are important in mathematics education, learners have shown considerable difficulties in understanding and verifying the fundamentals concepts of geometric reasoning used to prove midpoint theorem. The analysis of the responses reveals the existence of multiple cognitive barriers particularly across Newman's error categories. signifying the determination for the identification of the specific points within the breakdown in the students' problem-solving process.

The midpoint theorem requires integration of conceptual understanding with measurement skills and logical reasoning. It serves as an excellent diagnostic tool for exploring these difficulties. Previous research has revealed the misconceptions about geometric relationships are determined (Clement & Battista, 1992); and that students often struggle to connect algebraic representations with geometric principles (Duval, 2006). However, in the specific context of geometric verification work, few studies have systematically classified these errors using a comprehensive framework related to Newman's Error Analysis. The problem occurs and compounded by conventional teaching methods. These conventional methods happen to prioritize procedural knowledge over conceptual understanding, which results the difficulties of students to convert theoretical concepts into experimental verification. This research addresses this gap by applying Newman's Error Analysis to identify specific cognitive barriers that obstruct the students' ability to verify the midpoint theorem experimentally and apply the concept to prove other similar type of proof. To addresses the analysis of errors, the following objectives were used.

Objectives

This study has the following objectives:

1. To explore the students' errors for the experimental verification of the 'Midpoint theorem.
2. To explore the dimensions of students' errors in geometrical reasoning based on cognitive and mathematical languages

For fulfilling these objectives, this study provides a deeper insight into the students' geometric analytical reasoning and practical strategies to improve the instructional approaches in the use of appropriate measurement for the verification of geometric theorems.

Theoretical Framework

Anne Newman's error analysis framework, deals a structured methodology for diagnosing the complications in mathematical problem solving (Newman, 1977; 1983). According to Newman (1977), the successful problem solving skills is categorized into five sequential steps with difficulties at any stages potentially preventing the progression to subsequent steps (White, 2005). The five hierarchical stages of Newman's procedure are:

1. ***Reading***: The ability to recognize and decode words and symbols in the mathematical problem. This comprises elementary literacy and knowledge of mathematical terminology (Newman, 1977).
2. ***Comprehension***: In the comprehension error, the understanding of the meaning of the problem as a whole. At this stage, students must understand the problem asking them to find or verify (Prakitipong & Nakamura, 2006).
3. ***Transformation***: Transformation errors is related to understanding the problem into appropriate mathematical operations. This involves selecting appropriate approaches, formulae, or procedures to solve the problem (Clements, 1980).
4. ***Process Skills***: Process skills errors are related to the performing the mathematical operations correctly and identified in the transformation stage. This includes computational accuracy and procedural fluency (White, 2010).
5. ***Encoding***: Expressing the mathematical solution in an appropriate form as required by the problem. This involves articulating conclusions and expressing answers in standard measured language (Clarkson, 1991).

According to Newman (1983), it was found that 70 percent of mathematical errors found in the initial three stages in the comparison to later two stages. Further, Clements (1980) and Clarkson (1991) argued the established conceptual skills in mathematics education, indicating language comprehension and problem interpretation. The NEA framework holds up most importance for analyzing geometrical problem solving, where conceptual understanding should be taught into actual measurement and verification process. White (2010) stated that NEA helps and identify the students' difficulties in different branches of the solution of the problems from linguistics comprehension, conceptual understanding, procedural knowledge, or communication skills, approving for more focused remediation. In this context, we were trying to explore the nature of students' errors for the experimental verification of the geometrical statement related to Newman's Error analysis procedures. The students' error was identified thorough the procedures to provide experimental verification of geometric statements. The students' errors were analyzed by taking a statement of the most important theorem 'Midpoint theorem '.

Question No 1. Verify experimentally the line formed by joining two middle points of two sides of a triangle is half of the third side. (Two triangles with different measurements are needed)

Analysis procedures: In this process of analyzing the errors of students, researchers were collected the answer sheet of the randomly selected sixteen students from a community school of Kathmandu valley and analyzed their process of proving the theorem experimentally. The answer sheets of sixteen students were categorize in to four groups according to the four patterns of errors: Reading and Comprehension, Transformation, Process skills and Encoding errors. Students' answer sheets were categorized according to these types of errors. Among each category, answer sheets of four students were randomly selected from each of the four categories and named the codes as S_1, S_2, S_3 and S_4. The answer sheet of each of the four students were analyzed as below: The analysis of error by S_1 in the solution of the given problem was identified with the answer sheet of student S_1 was analyzed by using Newman's Error Analysis procedure.

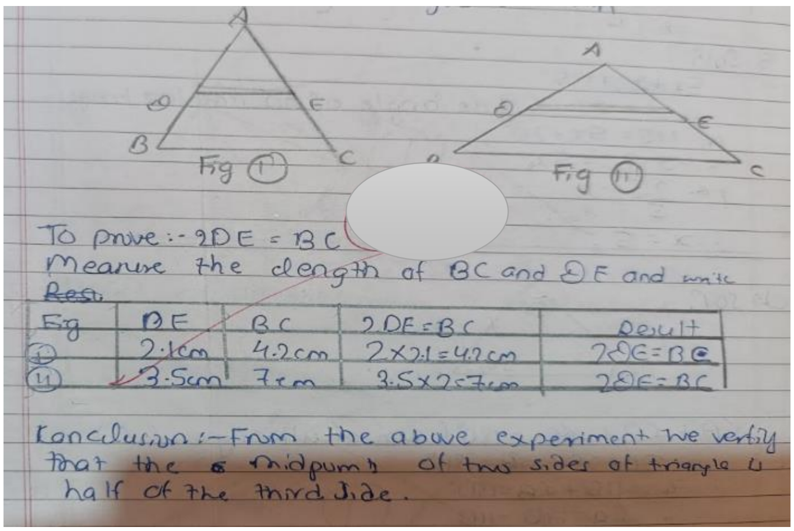
***Reading*:** The student demonstrates adequate reading ability as he/she correctly set up the problem to prove 2DE = BC. Student $\mathbf{S_1}$ drawn appropriate figures and established a clear structure for their verification process.

Comprehension: The student shows good comprehension of the geometric concept being tested. They understood that measurements needed to be taken and compared to verify the relationship between DE and BC.

***Transformation*:** The transformation phase appears sound as the student correctly organized data in a tabular format and identified the appropriate mathematical operation (multiplication by 2) to verify the relationship as given in the Figure 1.

Figure 1.

Answer Sheet of Student S_1 for the statement



***Process Skills*:** The student's calculations are accurate. They correctly determined that 2 × 3.5cm = 7cm, showing proficiency in basic arithmetic required for this proof.

Encoding: The student comes across the difficulty at the encoding stage. While his/her numerical work aligns with the conclusion that 2 × DE = BC, his/her written conclusion is incomplete. The statement "From the above experiment, we verified that the line joining the middle points of two sides of the triangle is half of the third side" contains a critical omission. The missing term should likely be "line joining the midpoints" or similar, as the midpoint theorem found that a line connecting the midpoints of two sides of a triangle is parallel to and half the length of the third side.

From the analysis of answer sheet of students S_1 the following summary was drawn.

Summary of the Answer Sheet of Student S_1

Student S_1 exhibited strong mathematical skills across the initial four phases of NEA and fails in the final encoding phase, where they need to clear geometrical concept in specific mathematical language. The incomplete conclusion informed that the student understands the mathematical relationship but failed to express theorem verbally.

Newman Error Analysis for Student S_2

Analyzing the answer sheet of Student S_2 using the NEA framework for the task of verifying that ' The line formed by joining two midpoints of triangle sides equals half the length of the third side':

Reading Error

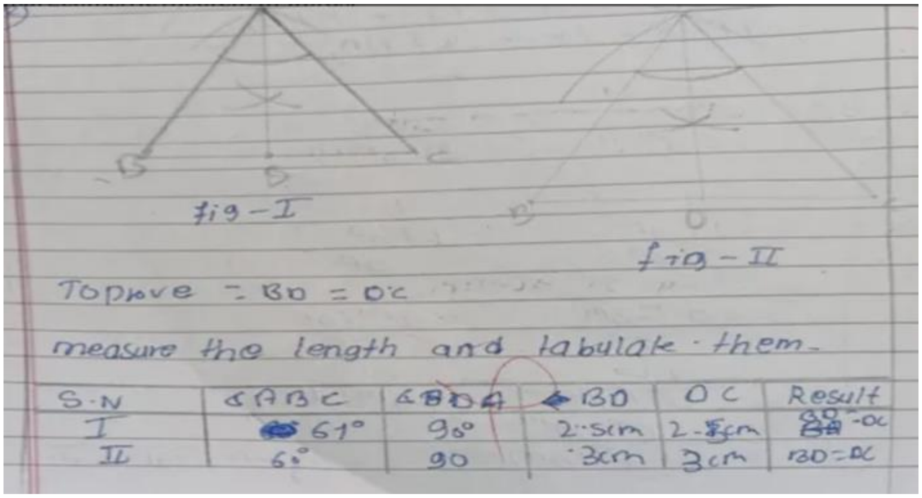
In the analysis of reading errors of students S_2, it was found that the student S_2 misinterprets the problem statement. The task requires verifying that a line connecting midpoints equals half of the third side, but his/her proof statement reads, "To prove: BO = OC" which doesn't directly address the midpoint relationship as required. This indicates a reading error that the student may not have completely understand what was to verify.

Comprehension Error

In the process of analyzing comprehension error of students, it was found that the student demonstrates partial comprehension of geometric concepts. He/she drew two triangles (Fig-I and Fig-II) and constructed a table, showing that he/she recognized the need for experimental verification with different measurements. However, he/she does not explicitly identify or mark midpoints in the diagrams, indicating incomplete comprehension of the midpoint theorem under verification.

Figure 2.

Answer Sheet of student S_2 for the statement



Transformation

There are significant transformation errors. The student measures angles (61°, 90° in Fig-I and 60°, 90° in Fig-II) rather than focusing on the key lengths needed to verify the midpoint theorem. This suggests the student transformed the problem into an angle-based verification rather than a length-based verification

Process Error

Process skill errors are evident in the data collection. The table includes columns for angles (∠ABC, ∠ABD) and lengths (BO, OC), but doesn't properly organise the measurements needed to verify the midpoint relationship. The recorded measurements (2.5cm, 2.5cm in row I and 3cm, 3cm in row II) don't relate to the required verification.

Encoding

The student's results column contains "BO=OC" which doesn't properly encode the midpoint theorem. The proper encoding would verify that a line connecting midpoints equals half the third side, but instead, the student appears to be verifying that BO equals OC, which is a different geometric relationship altogether.

Summary of the Answer Sheet of Student S_2

Student S_2 committed errors at all steps of the NEA framework, but particularly in the reading and transformation stages. They misinterpreted the task, measuring inappropriate quantities (angles instead of focusing on midpoints), and failed to properly encode the midpoint theorem relationship. The student appears to be verifying segment equality (BO=OC) rather than the relationship between a line connecting midpoints and half the length of the third side.

Newman Error Analysis for Student S_3

Analyzing the answer sheet of Student S_3 using Newman's Error Analysis framework for the task of verifying that the line formed by joining two midpoints of the sides of a triangle equals to the half the length of the third side:

Reading

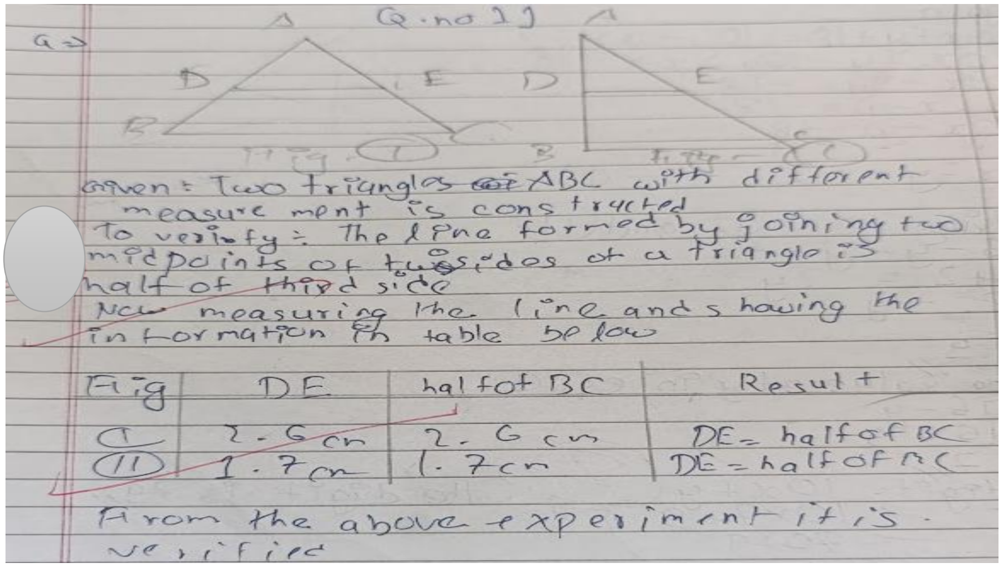
Student S_3 demonstrates adequate reading ability by correctly interpreting the task. The student's statement "Given: two triangles ABC with different measurements" and "to verify the line formed by joining two middle points of two sides of a triangle is half of third side" indicates they understood what was being asked.

Comprehension

The student shows good comprehension of the geometric concept being tested. They correctly identify that they need to measure the segments DE (the line connecting midpoints) and BC (the third side), and compare their lengths to verify the relationship.

Figure 3.

Answer Sheet of Student S3 for the given statement



Transformation

The transformation is appropriate as the student sets up a table with columns for "DE," "half of BC," and "Result" showing they understood the need to compare DE with half of BC to verify the relationship.

Process Skills

The process skills are sound. The student measured DE = 7.5cm and half of BC = 7.5cm in the first triangle, and DE = 7.7cm and half of BC = 7.7cm in the second triangle. The measurements are consistent with the relationship being verified.

Encoding

Student S_3 made a minor encoding error. The student wrote the concluding sentence “ *from the above experiment it is verified*” is incomplete. The student has not completely expressed what actually was verified, though the structure and process sustenance the proof.

In summary, Student S_3 exhibits resilient understanding through the initial four phases of NEA, and only a minor encoding error. The student appropriately framed the experiment, gathered essential measurements from two different triangles, and structured the data to display that DE equals half of BC in both, efficiently substantiating the given experimental verification.

The student's effort is considerably more aligned with the work requirements compared to the student S_2 as they properly concentrated on the midpoint relationship instead of angles or other unnecessary measurements.

Newman Error Analysis for Student S_4

Analyzing the answer sheet of Student S_4 using Newman's Error Analysis framework for the task of verifying that ' *The line formed by joining two midpoints of triangle sides equals half the length of the third side'*:

Reading

Student S_4 demonstrates good reading comprehension of the task. They clearly state, "To verify: The line formed by joining two midpoints of two sides of a triangle is half of the third side," accurately reflecting the given problem statement.

Comprehension

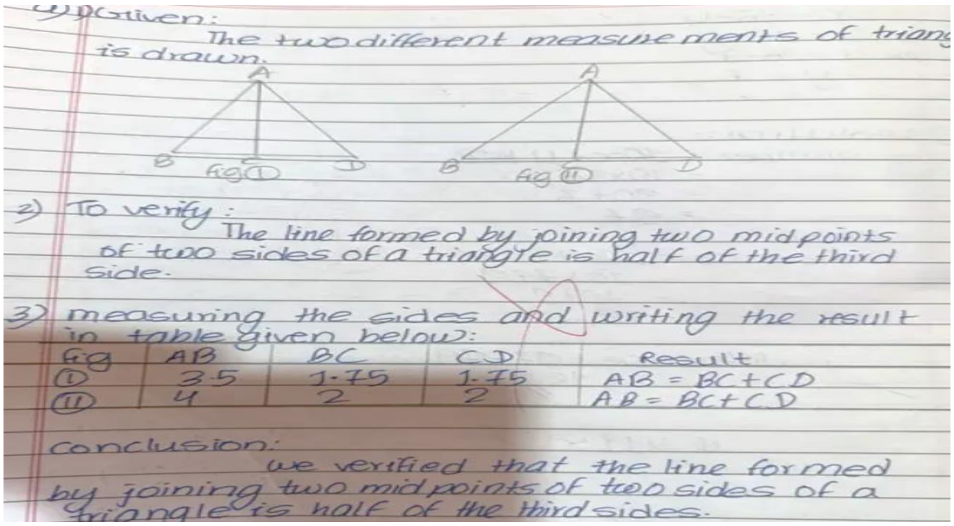
There is a serious comprehension error. Despite correctly stating the task, the student seems to misunderstand the geometric relationship being verified. Instead of comparing the midpoint connector line with half the third side, their table and results show they're attempting to verify that AB = BC + CD, which is completely unrelated to the midpoint theorem.

Transformation

The transformation is significantly flawed. The student has drawn appropriate triangles with labelled points, but their table measures AB, BC, and CD rather than measuring the line connecting midpoints and comparing it to half the third side. This indicates they transformed the problem incorrectly into verifying an incorrect relationship.

Figure 4.

Answer Sheet of Student S_4 for the given statement



4) Given:
The two different measurements of triang
is drawn.

A B D fig(I)

A B D fig(II)

2) To verify:
The line formed by joining two midpoints of two sides of a triangle is half of the third side.

3) measuring the sides and writing the result in table given below:

fig	AB	BC	CD	Result
(I)	3.5	1.75	1.75	AB = BC+CD
(II)	4	2	2	AB = BC+CD

conclusion:
we verified that the line formed by joining two midpoints of two sides of a triangle is half of the third sides.

Process Skills

The student's process skills cannot be properly evaluated regarding the intended task since they're solving a different problem altogether. Their arithmetic appears correct for their misconceived problem (AB = BC + CD), but this is irrelevant to the midpoint theorem verification requested.

Encoding

There is a substantial encoding error. Despite their work not supporting the midpoint theorem, the student incorrectly concludes: "Student S_4 verified that the line formed by joining two midpoints of two sides of a triangle is half of the third side." This conclusion is not supported by their data or calculations.

The student S4 made error in basic comprehension, which conquers through the entire solution process. However, S4 properly read and intrepid the problem, they failed to understand what midpoints are and verification process of the theorem. They carried out the verification process of an improper relationship (i.e. AB =BC+ CD), and then incorrectly argued the theorem have verified. This showed a clear gap between understanding of the problem and the geometrical notion to be verified. The critical error appeared in comprehension stage.

Conclusion

Based on the analysis of the data, we found that students have made several errors in each phase of NEA framework in comprehension and verification of geometrical problems. The major error patterns were: Reading comprehension were diverse significantly among the students. Most of the students were able to read the problem statement excluding S_2, who misread what accurately was asked to verify. Conceptual understanding was found signified the most remarkable gap. Only S_1 and S_3 demonstrated ample comprehension of the midpoint theorem, while S_2 and S_4 exposed fundamental misunderstandings of the geometric relationship. Transformation errors were established, particularly in S_2 and S_4, who set up inappropriate measurements (angles instead of lengths for S_2, and incorrect segment relationships for S_4). Process skills were usually adequate when the learners understood what to measure, but irrelevant when the problem was mistranslated. Encoding difficulties appeared across all the participants to varying degrees. Even the students who understood the concept (S_1 and S_3) seemed experiencing a trouble in fully articulating the conclusion.

The above investigation reveals a hierarchy of understanding among four participants: **S_3** performed the highest understanding of the midpoint theorem and appropriate experimental verification using two different triangles. **S_1** presented a good understanding but resulted in an imperfect conclusion statement. **S_2** demonstrated misconceptions regarding the stuff that required verification, focusing on angles than the midpoint relationship. S_4 produced the most significant errors having gross misunderstanding of the midpoint concept, verifying an irrelevant relationship such as (AB = BC+ CD) while the claiming and verified the midpoint theorem. Thus, the severe learning barrier appears to occur at the comprehension level despite the participants' reading capability. Most of the students were unable to understand the notion of midpoints and the process of verification of the theorem. This demands that the instructional interventions focus on enhancing the conceptual skill of geometric relationships prior to shifting to the verification tasks. For any prospective teaching activities, the concerned ought to emphasize the clear identification of the midpoints in diagrams, precisely establishing the relationship between the midpoint connector and the third side, and providing scaffolded guidance for properly expressing the geometric conclusions.

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